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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Path Loss Expressions for a Radio Link on a Rough Spherical Surface

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ABSTRACT

Expressions for calculating the path loss of a radio link are presented in a form suitable for use on a digital computer. The expressions apply to a radio link utilizing a vertically polarized signal and operating near a rough spherical surface such as that of the moon.

Three forms of path loss expressions are presented - one for each of the possible transmission regions in which a receiving antenna could be located.

1. Diffuse reflection region - near the transmitting antenna.

2. Specular reflection region - beyond region 1 and extending to near the horizon.

3. Diffraction region - near the horizon and beyond.

The three path loss expressions are based on three types of analysis. The first, diffuse reflection region, is based on the analysis of a simple free space path; the second, specular reflection region, is based on a geometric optics analysis; and the third, diffraction region, is based on Bremmer's residue series analysis of propagation.

Results of path loss calculations using the expressions presented are plotted for the following two Apollo lunar surface radio links:

1) Lunar Module - Extra Vehicular Astronaut (LM-EVA)

2) Extra Vehicular Astronaut (1) - Extra Vehicular Astronaut (2) (EVA-EVA).

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TECHNICAL MEMORANDUM

I. INTRODUCTION

In reference [1] Schmid presented expressions for predicting the path loss for a radio link operating near a rough spherical surface. Schmid's analysis has been extended by the use of Bremmer's [2] residue series analysis for calculating the path loss in the region immediately beyond the specular reflection region, and also by a general analysis of the link geometry. Expressions have been derived that permit implementing the revised path loss expressions in a digital computer program to provide rapid and, it is believed, accurate calculations.

Results of path loss calculations using the revised expressions summarized in Table I are compared in figures 1-3 with those results predicted by Schmid. The comparison shows the following:

1. The approximation for the grazing angle

$$\Theta_g \doteq \tan^{-1}\left[\left(h_1 + h_2\right)/ R_{sd}\right] .$$

used by Schmid is valid for both the LM-EVA and the EVA-EVA case over the entire range of their specular reflection regions. The lunar surface reflection factor $\left(\Gamma_r = \Gamma_p \rho_{rms} \mathrm{Div}\right)$ calculated here and shown in figure 1 is equal to that plotted by Schmid.

2. The approximation for the phase delay

$$\phi \simeq \Pi + (4\Pi h_1 h_2)/\lambda R_{sd}$$

of the reflected signal used by Schmid for the EVA-EVA case is valid over the entire specular reflection region. The path loss calculated here shown in figure 2 is equal to that plotted by Schmid.

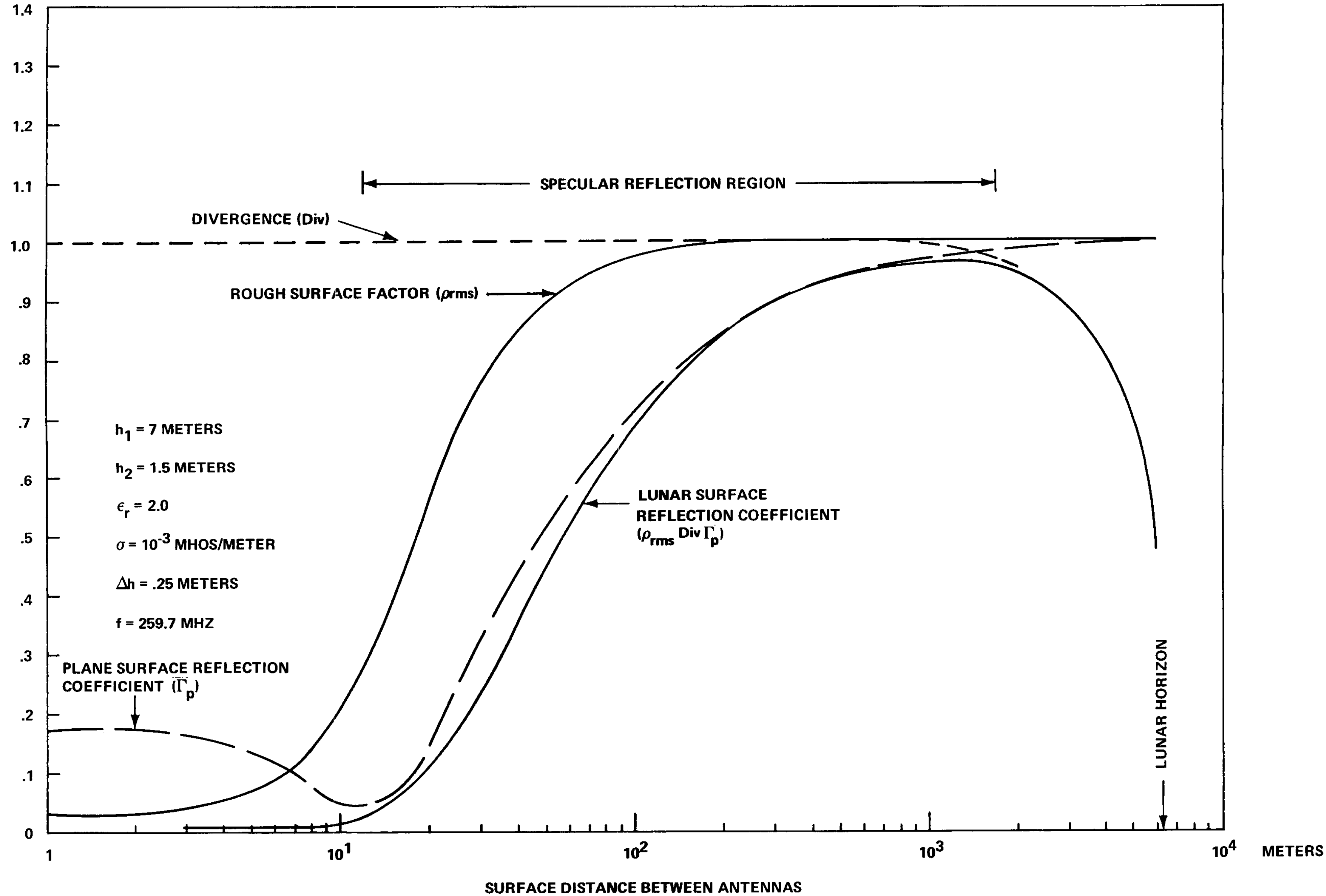


FIGURE 1 - LUNAR SURFACE REFLECTION COEFFICIENT AND ITS COMPONENTS
(CURRENT RESULTS EQUAL TO SCHMID'S RESULTS IN REFERENCE[1])

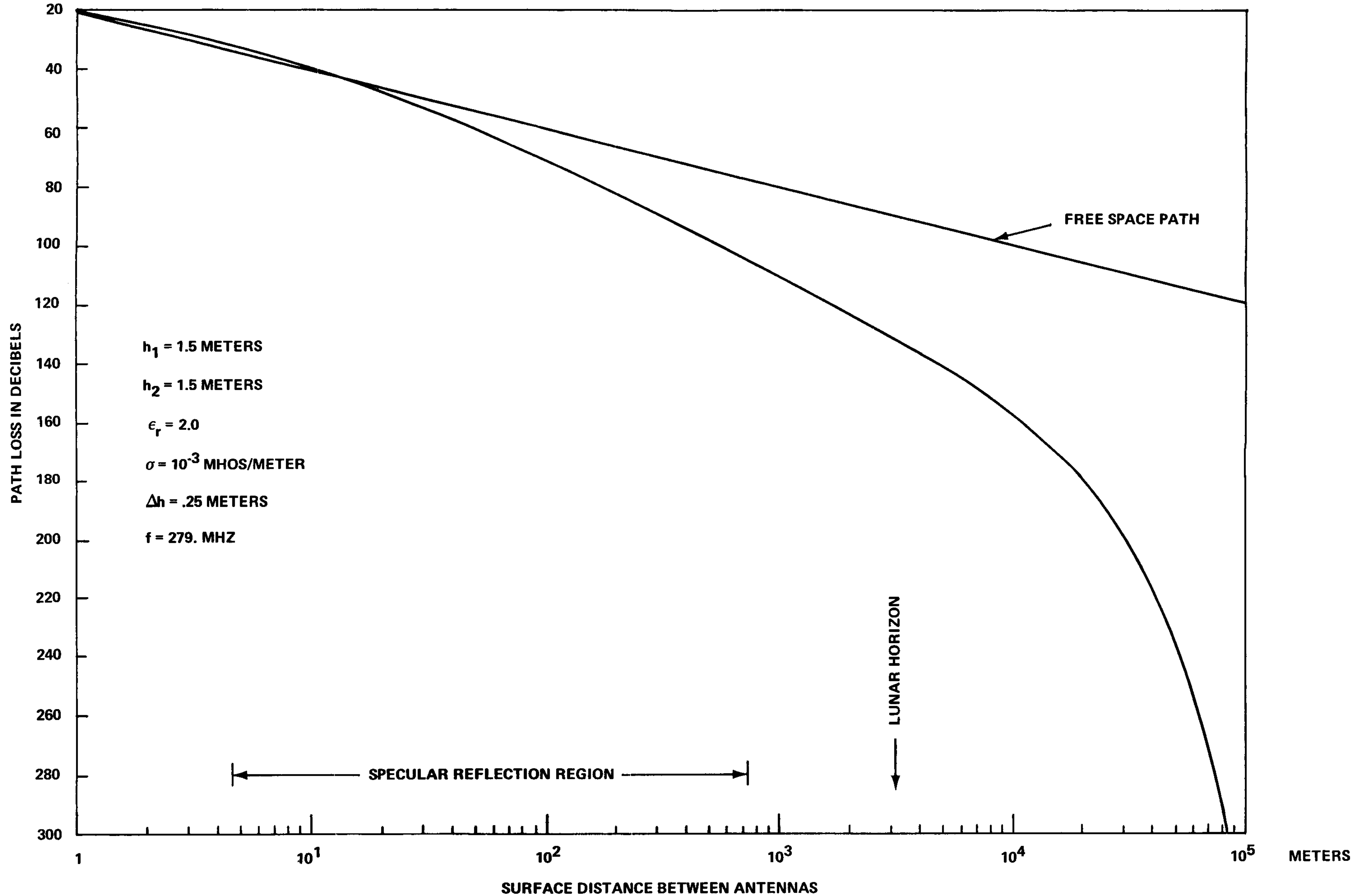


FIGURE 2 - PATH LOSS FOR EVA-EVA CASE (CURRENT RESULTS EQUAL TO SCHMID'S RESULTS IN REFERENCE [1])

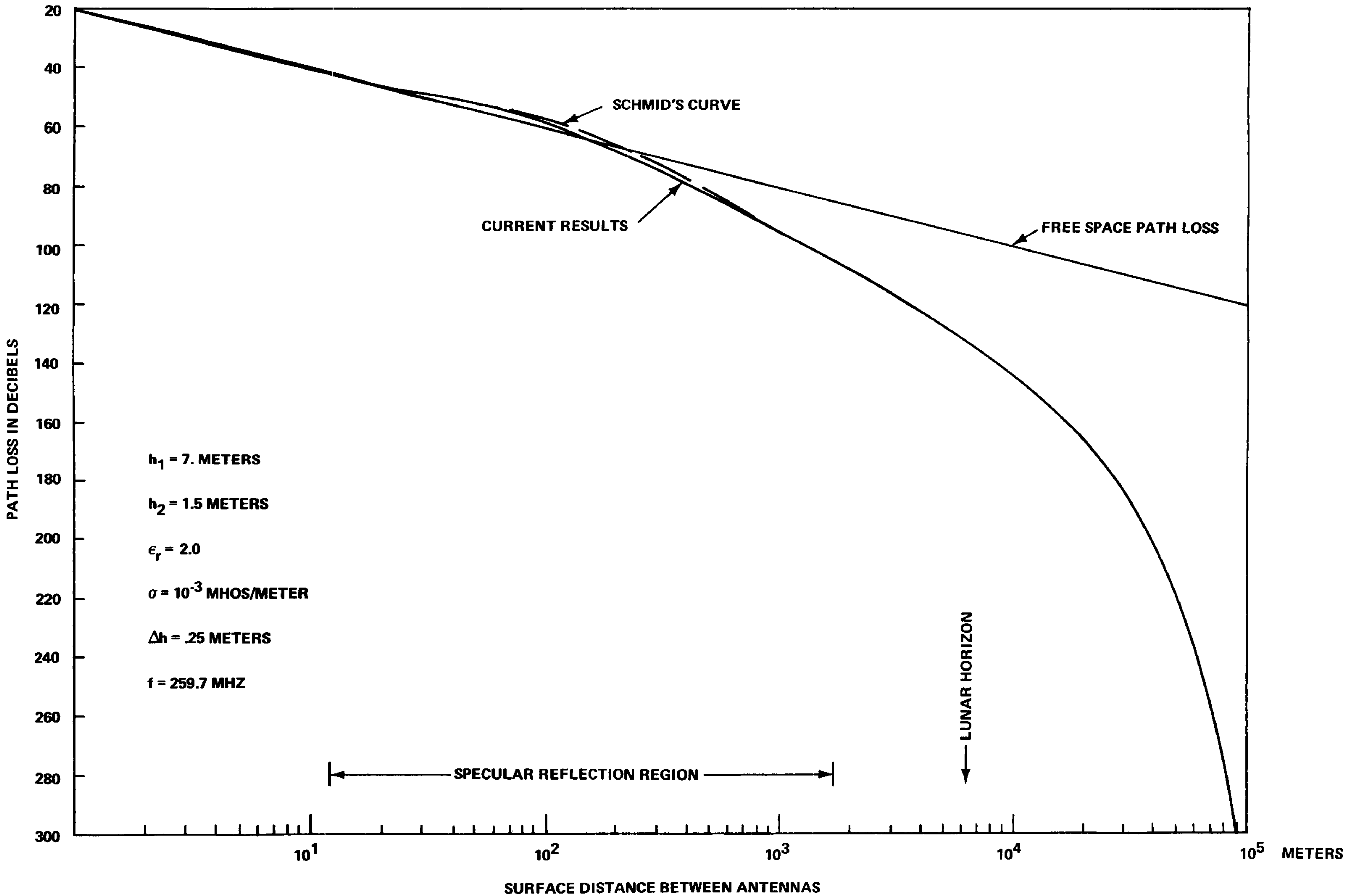


FIGURE 3 - COMPARISON OF CURRENT RESULTS WITH RESULTS PRESENTED BY SCHMID IN REFERENCE [1] FOR LM-EVA CASE

3. The use of the approximation for the phase delay

$$\phi \simeq \Pi + \Pi \left[\frac{h_1^2 + 2h_1h_2 + h_2^2}{\lambda R_{sd}} \right]$$

of the reflected signal used by Schmid for the LM-EVA case results in a lesser path loss than if the more accurate

$$\phi = \Pi + \frac{2\Pi\delta_{path}}{\lambda}$$

δ_{path} = difference in length of the direct and reflected paths calculated from the link geometry

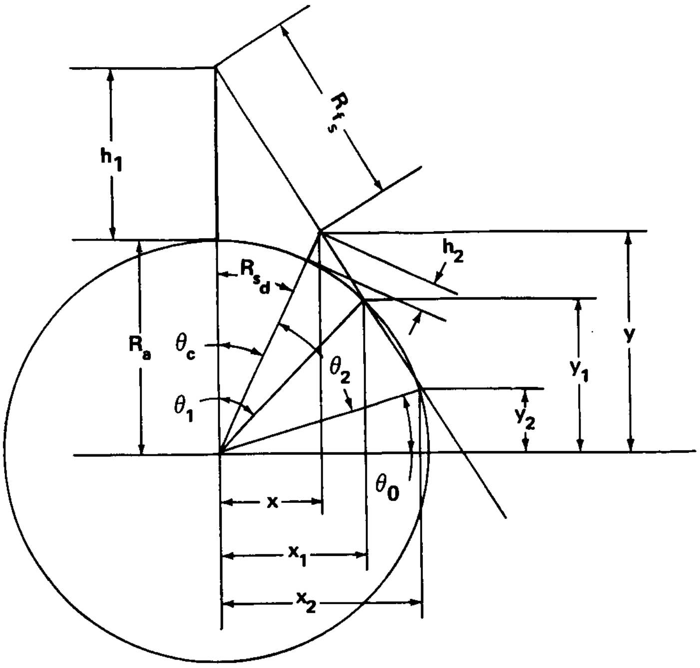
expression for the phase delay is used. Figure 3 shows this difference in the calculated path losses.

II. ANALYSIS OF THE PROBLEM

For analysis the region extending from a transmitting antenna, located on a rough spherical surface, to a point below the horizon can conveniently be divided into three regions. These regions are defined as follows:

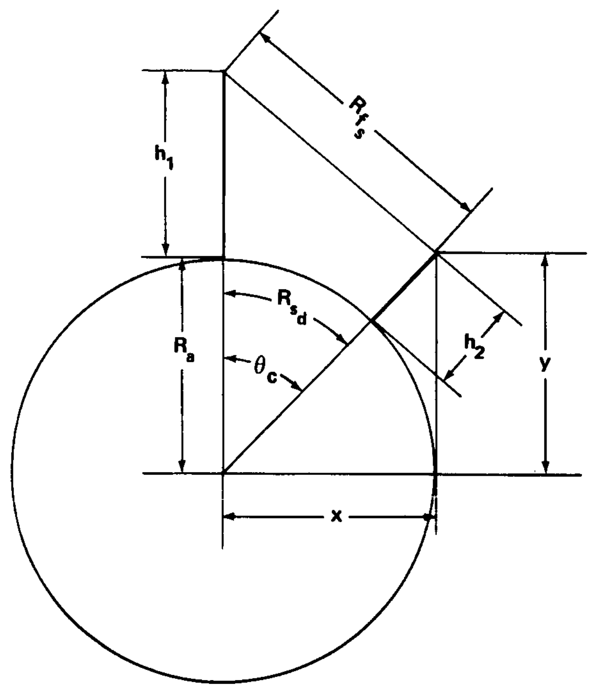
1. Diffuse Reflection Region
2. Specular Reflection Region
3. Diffraction Region

The basis for defining these regions lies in the character of the radio signal that exists at the receiving antenna. If there exists a line of sight path between the antennas, then reflected and direct signals will be present at the receiving antenna. Since the contribution to the received signal will be significant if the reflection from the surface is specular and negligible (by definition) if diffuse, the boundary between regions (1) and (2) above is calculated from the Rayleigh criterion which is a function of the carrier signal wavelength, the grazing angle of the reflected signal, and the roughness of the spherical surface. If no line of sight path exists between the antennas it is assumed for this analysis that the only process by which a radio wave can be propagated to the receiving antenna is by the diffraction phenomenon; therefore, region (3) above is defined as that region beyond which the specular reflection analysis is no longer valid.



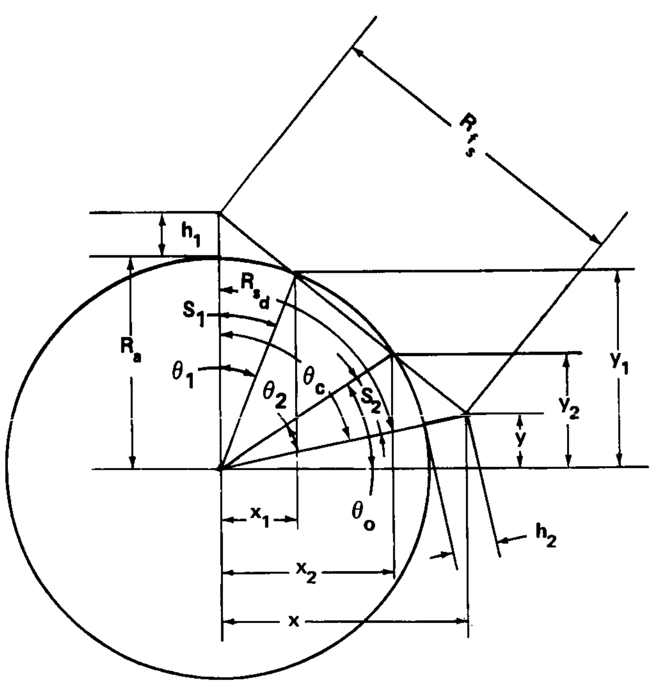
CASE a

FIGURE 4a



CASE b

FIGURE 4b



CASE c

FIGURE 4c

FIGURE 4 POSSIBLE CONFIGURATIONS OF RADIO LINK

The model of the transmission path that will be used in the following discussion is shown in figure 4; it consists of two antennas positioned h_1 and h_2 meters, respectively, above the lunar surface - separated by a surface distance of R_{sd} meters.

Since mathematical models have been derived for predicting the signal received in each of the regions defined above, our problem in calculating path loss for a given radio link, therefore, reduces to simply determining the region in which our receiving antenna is located.

III. EXISTENCE OF LINE OF SIGHT PATH DETERMINED

If the surface is approximated by a smooth sphere, then three possible configurations of the surface radio link are shown in figure 4. These configurations are the following:

a) The straight line connecting the two antennas when extended intersects the surface of a sphere that has a radius R_a.

b) The straight line connecting the two antennas does not intersect the surface of a sphere that has a radius R_a.

c) The straight line connecting the two antennas intersects the surface of a sphere that has a radius R_a; this intersection exists between the two antennas.

To determine which of the above configurations applies to a given combination of antenna heights and link lengths, the locations if any, of intersections between the straight line connecting the two antennas and a circle with a radius R_a are calculated.

Placing the origin of a rectangular coordinate system at the center of a circle, the circle is described by

$$x^2 + y^2 = R_a^2 \qquad (1)$$

R_a = Radius in meters

and the straight line connecting the two antennas is described by

$$y = mx + b \tag{2}$$

b = The y axis intercept = $R_a + h_1$ meters

h_1 = Height of antenna (1) in meters

m = Slope of line = $\frac{y_i - y_j}{x_i - x_j}$

To find the slope m, the coordinates of antenna (2) are calculated and defined as X, Y. (See figure 4).

$$\theta_c = R_{sd}/R_a \quad \text{= the central angle defined by the antenna separation } R_{sd} \tag{3}$$

$$X = (R_a + h_2) \cos \left(\frac{\pi}{2} - \theta_c\right) = (R_a + h_2) \sin \theta_c \tag{4}$$

$$Y = (R_a + h_2) \sin \left(\frac{\pi}{2} - \theta_c\right) = (R_a + h_2) \cos \theta_c \tag{5}$$

h_2 = height of antenna (2) in meters

then

$$m = \frac{(R_a + h_1) - (R_a + h_2) \cos \theta_c}{0 - (R_a + h_2) \sin \theta_c} \tag{6}$$

$$= \frac{(R_a + h_2) \cos \theta_c - (R_a + h_1)}{(R_a + h_2) \sin \theta_c}$$

Now substituting equation 2 into 1 and solving for x gives

$$x^2 + (mx + b)^2 = R_a^2$$

then

$$x = \frac{\frac{-2mb}{1+m^2} \pm \left[\left(\frac{2mb}{1+m^2}\right)^2 - \frac{4\left(b^2-R_a^2\right)}{(1+m^2)}\right]^{1/2}}{2}$$

$$x_1 = \frac{-mb}{1+m^2} + \left[\left(\frac{mb}{1+m^2}\right)^2 - \left(\frac{b^2-R_a^2}{1+m^2}\right)\right]^{1/2} \quad (7)$$

$$y_1 = mx_1+b$$

$$x_2 = \frac{-mb}{1+m^2} - \left[\left(\frac{mb}{1+m^2}\right)^2 - \left(\frac{b^2-R_a^2}{1+m^2}\right)\right]^{1/2} \quad (8)$$

$$y_2 = mx_2+b$$

The figure 4 configuration that applies to a given set of antenna heights and link lengths is therefore determined from equation (7) and (8) as follows:

Let $x_1 \leq x_2$ and both are positive if $h_1 \geq h_2$

1. If x_1 and x_2 are real and $x_1 > X$ then the configuration of figure 4a applies to the given radio link.
2. If x_1 and x_2 are real and $x_1 < X$ then the configuration of figure 4c applies to the given radio link.
3. If x_1 and x_2 are imaginary, then, the configuration of 4b applies to the given radio link.

An approximation that is useful for this smooth surface case defines the surface distance to the horizon; this is expressed as follows:

$$R_{hor} = \left(2R_a h\right)^{1/2} \quad (9)$$

h = Height of the given antenna

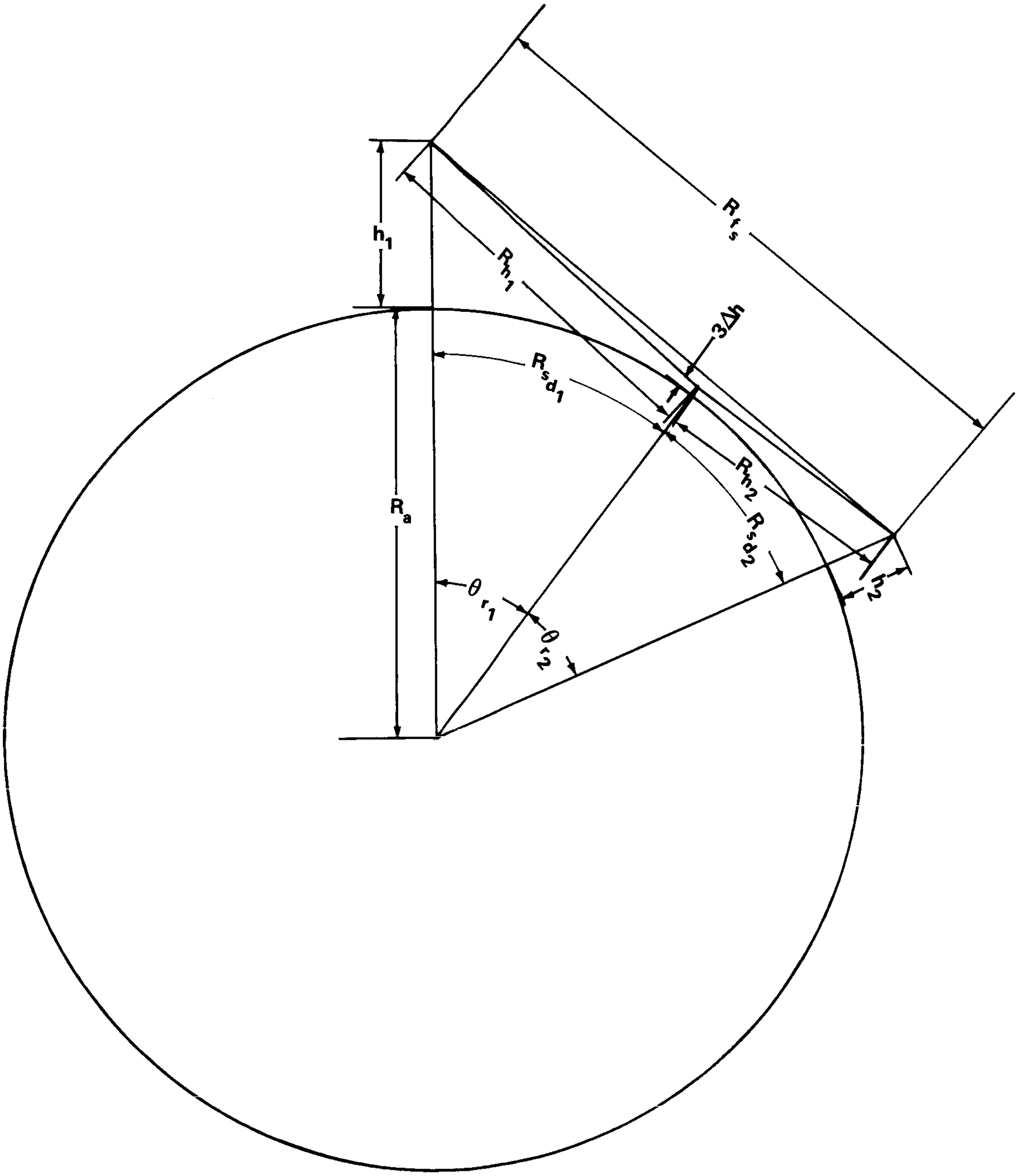


FIGURE 5 – INTERFERENCE OF LINE OF SIGHT PATH BY ROUGH SURFACE

The maximum surface distance between antennas that still permits a line of sight path to exist between them is therefore defined by

$$R_{sd\ MAX\ (smooth)} = \left(2R_a h_1\right)^{1/2} + \left(2R_a h_2\right)^{1/2} \qquad (10)$$

For a surface that is rough but essentially spherical in shape, the configuration of figure 5 must be considered. The standard deviation of the heights in the surface irregularities is defined as Δh. Assuming that it was determined by the analysis for the smooth surface above that a line of sight path exists for a given radio link, we must now determine if this path clears the rough surface.

by the law of sines

$$\frac{R_{h_1}}{\sin\Theta_{r1}} = \frac{R_a+h_1}{\sin\frac{\pi}{2}}, \quad \frac{R_{h_2}}{\sin\Theta_{r2}} = \frac{R_a+h_2}{\sin\frac{\pi}{2}} \qquad (11)$$

$$\Theta_{r1} = \sin^{-1}\left[\frac{R_{h1}}{R_a+h_1}\right], \quad \Theta_{r2} = \sin^{-1}\left[\frac{R_{h2}}{R_a+h_2}\right] \qquad (12)$$

since $R_{h1} = \left[\left(R_a+h_1\right)^2 - \left(R_a+3\Delta h\right)^2\right]^{1/2}$, $R_{h_2} = \left[\left(R_a+h_2\right)^2 - \left(R_a+3\Delta h\right)^2\right]^{1/2}$

the maximum surface distance between two antennas, located on a rough surface, that permits a line of sight path to exist between them is therefore given by

$$R_{sd\ MAX\ (rough)} = \left(\Theta_{r1}+\Theta_{r2}\right)R_a \qquad (13)$$

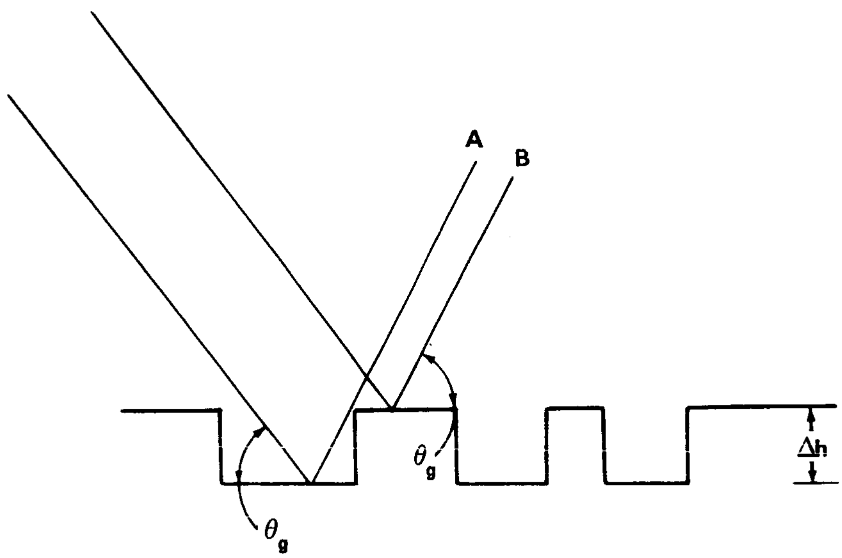


FIGURE 6 - REFLECTIONS FROM A ROUGH SURFACE

IV. PATH LOSS FOR THE REGION ABOVE THE HORIZON

a. Rayleigh Criterion

Assuming that it has been determined that a line of sight path exists between the two antennas of a given radio link operating near a rough spherical surface, we must then determine if the reflections from the surface arriving at the receiving antenna can be considered to be diffuse or specular. This distinction is provided by the Rayleigh [3] criterion.

A plane parallel wave after being reflected from a rough surface is comprised of at least two components as shown in figure 6. The Rayleigh relation simply indicates the phase difference or spread in phases between these reflected components as a function of the surface roughness (Δh) and the grazing angle (Θ_g) of the incident plane parallel wave. The "Rayleigh Criterion"

$$\Delta\phi = \frac{4\Pi\Delta h}{\lambda} \sin \Theta_g \text{ radians} \tag{14}$$

λ = wavelength of carrier signal

then, by setting $\Delta\phi = \frac{\Pi}{2}$ as a maximum acceptable spread in the phases of the reflected components [3], can be used to establish a constraint on the grazing angle (Θ_g) for our radio link.

$$\Delta\phi \leq \frac{\Pi}{2} \geq \frac{4\Pi\Delta h}{\lambda} \sin \Theta_g \tag{15}$$

$$\Theta_g \leq \sin^{-1} \left[\frac{\lambda}{8\Delta h}\right] \text{ radians} \tag{16}$$

Therefore, if relation (16) is satisfied the phase spread of the reflected components is less than $\frac{\Pi}{2}$ radians and the surface will be considered smooth, capable of supporting specular reflections; however, if this relation is not satisfied, the reflections will be considered diffuse. The calculation of the grazing angle will be discussed later.

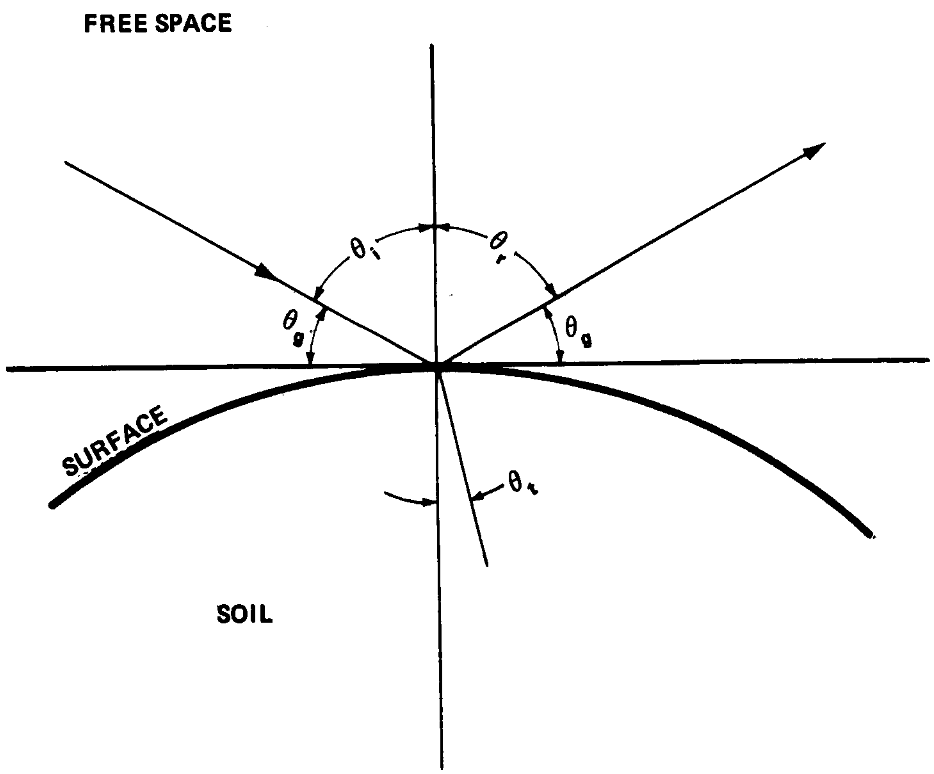


FIGURE 7 — INTERFACE GEOMETRY USED IN CALCULATING THE SURFACE COMPLEX REFLECTION COEFFICIENT

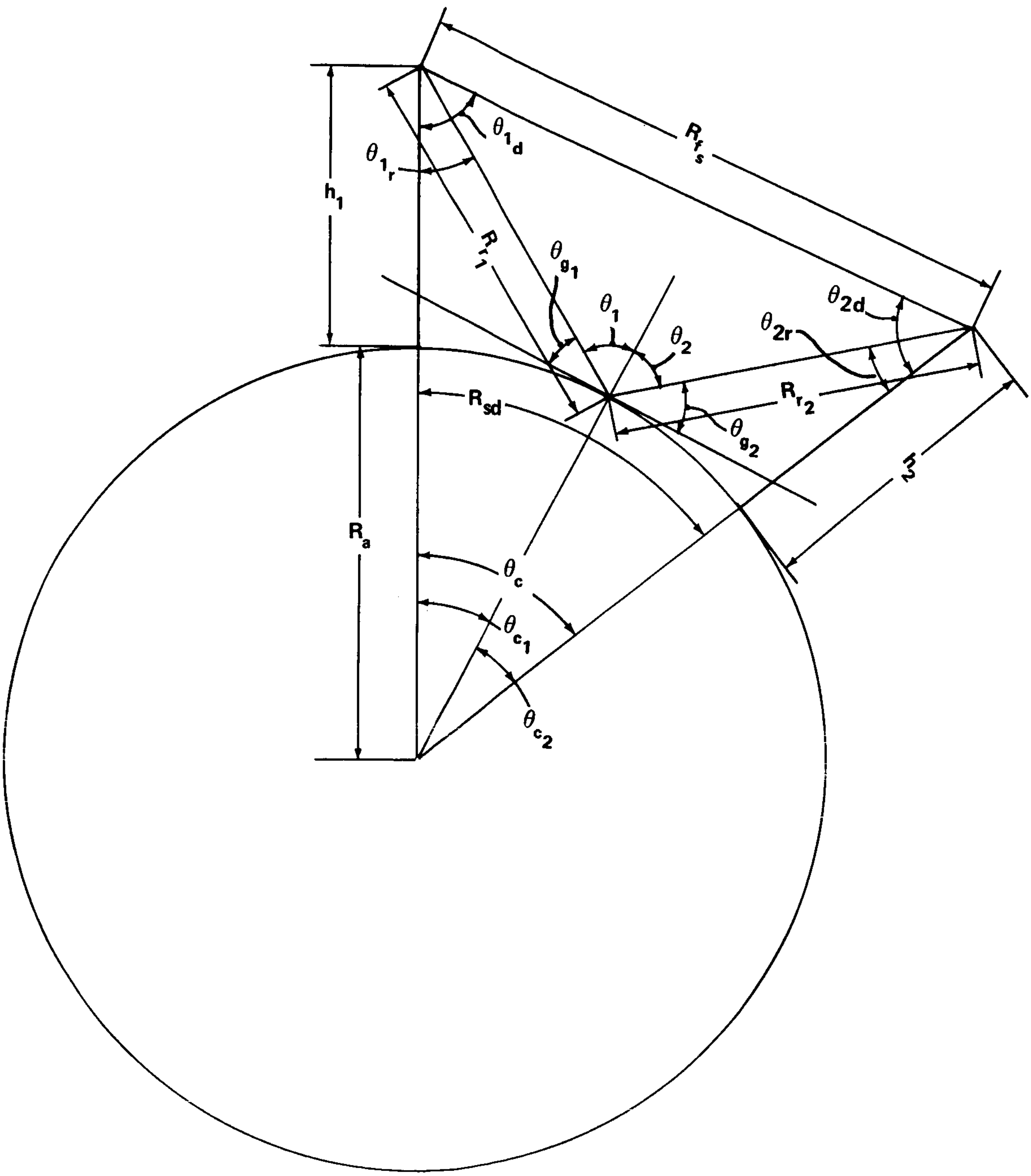


FIGURE 8 - GEOMETRY FOR REFLECTION REGION ASSUMING TRANSMISSION OVER A SMOOTH SPHERICAL SURFACE USING TWO DIPOLES

b. Path Loss for the Diffuse Reflection Region

Since surface reflections that are diffuse will contribute little to the signal received at the receiving antenna, the path loss for the region where relation (16) is not satisfied is closely approximated by the path loss of a free space path.

$$\text{Path loss (I)} = 20 \log_{10} R_{fskm} + 20 \log_{10} F + 32.45 \text{ db} \qquad (17)$$

R_{fskm} = Direct path distance between antennas in kilometers

F = Frequency of Carrier Signal in Mhz

C. The Grazing Angle Calculated

A wave incident on a surface at an angle Θ_i (see figure 7) will, if reflected at all, be reflected from that surface at an angle Θ_r where

$$\Theta_i = - \Theta_r \qquad (18)$$

if both angles are measured from a line drawn normal to the surface at the point on the surface where reflection occurs. The magnitude of the reflected signal is a function of this incident or reflection angle. In the discussion that follows the compliment of $|\Theta_i| = |\Theta_r|$ is defined as the grazing angle Θ_g. Direct calculation of this angle is not possible, however, an iterative technique will now be presented that works quite well.

Turning now to figure 8, we see that the following information is known:

1. The heights of the two antennas - h_1, h_2
2. The surface distance between the antennas - arc length R_{sd}
3. The fact that $|\Theta_{g1}| = |\Theta_{g2}|$
4. The angles Θ_{g1} and Θ_{g2} are measured from a line that is tangent to the surface at the point where reflection occurs.

Using the law of sines the following relation is obtained. (See figure 8).

$$\frac{R_a}{\sin\left[\Pi-\frac{\Pi}{2}-\Theta_{g1}-\Theta_{c1}\right]} = \frac{h_1+R_a}{\sin\left[\frac{\Pi}{2}+\Theta_{g1}\right]} \tag{19}$$

since

$$\sin\left[\frac{\Pi}{2} \pm \Theta\right] = \sin\frac{\Pi}{2}\cos\Theta \pm \cos\frac{\Pi}{2}\sin\Theta$$
$$= \cos\Theta$$

then

$$\frac{R_a}{\cos(\Theta_{g1}+\Theta_{c1})} = \frac{h_1+R_a}{\cos\Theta_{g1}}$$

$$\frac{\cos\Theta_{g1}\cos\Theta_{c1}-\sin\Theta_{g1}\sin\Theta_{c1}}{\cos\Theta_{g1}} = \frac{R_a}{h_1+R_a}$$

$$\cos\Theta_{c1} - \tan\Theta_{g1}\sin\Theta_{c1} = \frac{R_a}{h_1+R_a}$$

Then solving for Θ_{g1} gives

$$\Theta_{g1} = \tan^{-1}\left[\frac{\frac{R_a}{h_1+R_a} - \cos\Theta_{c1}}{-\sin\Theta_{c1}}\right] \tag{20}$$

and in a similar manner

$$\Theta_{g2} = \tan^{-1}\left[\frac{\frac{R_a}{h_2+R_a} - \cos\Theta_{c2}}{-\sin\Theta_{c2}}\right] \tag{21}$$

The location of the reflection point on the surface is unknown, consequently, Θ_{c1} and Θ_{c2} are unknown, which necessitates the use of the iteration process to find the combination of Θ_{c1} and Θ_{c2} that results in $\Theta_{g1} = \Theta_{g2}$ as defined by equations (20) and (21).

An iteration process that will perform quite well is as follows:

1. Set upper and lower bounds on Θ_{c1} that initially are contrained to (22)

$$\Theta_{c1}\ (\text{high}) = \Theta_c$$

$$\Theta_{c1}\ (\text{low}) = 0$$

2. Select a test Θ_{c1} by

$$\Theta_{c1i} = \Theta_{c1}\ (\text{low}) + \frac{j}{k}\,\Theta_{c1}\ (\text{high})$$

j and k are arbitrary integers used to increment Θ_{c1i}

3. Calculate Θ_{c2} by

$$\Theta_{c2} = \Theta_c - \Theta_{c1}$$

4. Calculate Θ_{g1} and Θ_{g2} by (20) and (21)

5. a. If $|\Theta_{g1} - \Theta_{g2}|$ is within the accuracy desired the solution is complete.

   b. If Θ_{g2} is negative or $\Theta_{g1} > \Theta_{g2}$ then store the magnitude of Θ_{c1} that was used and increment j.

   c. If $\Theta_{g1} < \Theta_{g2}$ then increment k, reset j to some low initial value, set Θ_{c1} (low) to the largest value of Θ_{c1i} used for which Θ_{g2} was negative or $\Theta_{g1} > \Theta_{g2}$, set Θ_{c1} (high) to the smallest value of Θ_{c1i} used for which $\Theta_{g1} < \Theta_{g2}$, and repeat steps 2, 3, 4 and 5.

The result of this iteration should give a grazing that satisfies the following:

$$0 \leq \Theta_g \leq \Pi/2 \qquad (23)$$

d. Reflection Coefficient for a Plane Surface

Using the grazing angle Θ_g as defined above and assuming the surface to be a flat plane (see figure 7) the reflection coefficient for a vertically polarized signal can be expressed by [3]

$$\overline{\Gamma}_p = \frac{\overline{\varepsilon}_c \sin \Theta_g - \left[\overline{\varepsilon}_c - \cos^2 \Theta_g\right]^{1/2}}{\overline{\varepsilon}_c \sin \Theta_g + \left[\overline{\varepsilon}_c - \cos^2 \Theta_g\right]^{1/2}} \qquad (24)$$

$$\overline{\varepsilon}_c = \varepsilon_r - j60\lambda\sigma$$

ε_r = Relative dielectric constant of the surface

λ = Wavelength of carrier signal in meters

σ = Conductivity of the surface in mhos/meter

and expressed in polar form by

$$\overline{\Gamma}_p = |\Gamma_p| \angle \Theta_\Gamma \qquad (25)$$

e. Divergence of a Spherical Surface

The reflection coefficient (Γ_p) defined in the previous section is derived for a plane surface; this factor can be used for a spherical surface also but it must be multiplied by the divergence factor (Div) of the surface. Divergence is strictly a function of the curvature of the surface from which a signal is reflected. For a perfectly smooth spherical surface the divergence factor is given by [3] (see figure 8)

$$\mathrm{Div} = \left\{1+\frac{2R_{r1}R_{r2}}{R_a\left[R_{r1}+R_{r2}\right]\sin\Theta_g}\left[1+\sin\Theta_g+\frac{2R_{r1}R_{r2}}{R_a(R_{r1}+R_{r2})}\right]\right\}^{-1/2} \tag{26}$$

The reflection coefficient then for a <u>smooth spherical</u> surface is

$$\bar{\Gamma}_s = \bar{\Gamma}_p \, \mathrm{Div} \tag{27}$$

It is important to note that equation (26) is only valid if the following constraint on the grazing angle is satisfied [4]:

$$\Theta_g \geq \tan^{-1}\left[\frac{\lambda}{2\Pi R_a}\right]^{1/3} \quad \text{radians} \tag{28}$$

f. Rough Surface Correction Factor

In addition to causing a dispersion of the phase of the incident wave, the rough surface also causes scattering of the incident wave. The factor by which the smooth surface reflection coefficient must be multiplied to account for this rough surface scattering is derived in reference [3] to be

$$\rho_{rms} = \left\{\exp\left[-\left(\frac{4\Pi\Delta h\sin\Theta_g}{\lambda}\right)^2\right]\right\}^{1/2} \tag{29}$$

The reflection coefficient then for a rough spherical surface is

$$\bar{\Gamma}_r = \rho_{rms} \, \mathrm{Div} \, \bar{\Gamma}_p \tag{30}$$

g. Brewster Angle Calculation

For a vertically polarized wave incident on a nonmagnetic surface, the reflected signal falls to zero if the incident angle is equal to the Brewster or polarizing angle defined by [5]:

$$\Theta_i = \tan^{-1}\left[\frac{\varepsilon_2}{\varepsilon_1}\right]^{1/2}$$

assuming that the medium above the lunar surface is free space, the Brewster angle for the lunar soil is given, in terms of the grazing angle, by

$$\Theta_g \text{ (Brewster)} = \frac{\Pi}{2} - \tan^{-1} (\varepsilon_r)^{1/2} \qquad (31)$$

h. Phase Delay of Reflected Signal

The specular reflection region is also referred to as the interference region because in this region the transmitted signal traveling the most direct path to the receiving antenna is in part cancelled, or reinforced , by the reflected signal that is delayed by some phase "Θ_{rf}". This phase delay is a function of two parameters, 1) the phase of the plane surface reflection coefficient "Θ_Γ", and 2) the difference in the path lengths of the direct and the reflected signal "δ_{path}".

The phase of the plane surface reflection coefficient is given by equation (25)

$$\bar{\Gamma}_p = |\Gamma_p| \angle \Theta_\Gamma$$

The phase delay caused by the difference in the path lengths is calculated as follows: (see figure 8)

After the grazing angle has been determined, the two central angles Θ_{c1} and Θ_{c2} are also known; therefore, the legs of the reflected path are by the law of sines:

$$\frac{R_{r1}}{\sin\Theta_{c1}} = \frac{R_a+h_1}{\sin\left(\frac{\Pi}{2}+\Theta_g\right)} \qquad (32)$$

$$R_{r1} = (R_a+h_1)\frac{\sin\Theta_{c1}}{\cos\Theta_g}$$

and likewise

$$R_{r2} = (R_a+h_2)\frac{\sin\Theta_{c2}}{\cos\Theta_g} \qquad (33)$$

by the law of cosines the direct path length is

$$R_{fs} = \left[(R_a+h_1)^2+(R_a+h_2)^2-2(R_a+h_1)(R_a+h_2)\cos\Theta_c\right]^{1/2} \qquad (34)$$

then the difference in the two path lengths is

$$\delta_{path} = R_{r1} + R_{r2} - R_{fs} \qquad (35)$$

the total phase delay for the reflected path then is

$$\Theta_{rf} = \Theta_\Gamma + \frac{2\Pi\delta_{path}}{\lambda} \qquad (36)$$

i. Antenna Gains

Assuming that the antennas used in the radio link are dipoles whose gain pattern can be expressed by

$$\text{Antenna Gain} = \frac{\cos\left(\frac{\Pi}{2}\cos\beta\right)}{\sin\beta} \qquad (37)$$

β = the angle measured between the single path and the local vertical the antenna gains can be calculated using the law of sines and the law of cosines (see figure 8)

a) For the direct path

$$(R_a+h_2)^2 = R_{fs}^2+(R_a+h_1)^2-2R_{fs}(R_a+h_1) \cos \Theta_{1d} \tag{38}$$

$$\cos \Theta_{1d} = \frac{\left(R_a+h_2\right)^2-R_{fs}^2 - \left(R_a+h_1\right)^2}{-2R_{fs} \ (R_a+h_1)}$$

$$\frac{R_a+h_2}{\sin\Theta_{1d}} = \frac{R_{fs}}{\sin\Theta_c}$$

$$\sin\Theta_{1d} = \frac{(R_a+h_2)}{R_{fs}} \sin\Theta_c \tag{39}$$

and likewise

$$\cos\Theta_{2d} = \frac{(R_a+h_1)^2 - R_{fs}^2 - (R_a+h_2)^2}{-2R_{fs}(R_a+h_2)} \tag{40}$$

$$\sin\Theta_{2d} = \frac{(R_a+h_1)}{R_{fs}} \sin\Theta_c \tag{41}$$

b) For the reflected path

$$R_a^2 = R_{r1}^2 + (R_a+h_1)^2 - 2R_{r1}(R_a+h_1) \cos \Theta_{1r}$$

$$\cos \Theta_{1r} = \frac{R_a^2 - R_{r1}^2 - (R_a+h_1)^2}{-2R_{r1}(R_a+h_1)} \tag{42}$$

$$\frac{R_a}{\sin\theta_{1r}} = \frac{R_a+h_1}{\sin\left(\frac{\Pi}{2}+\theta_g\right)} = \frac{R_a+h_1}{\cos\theta_g}$$

$$\sin\theta_{1r} = \frac{R_a}{R_a+h_1}\cos\theta_g \tag{43}$$

and likewise

$$\cos\theta_{2r} = \frac{R_a^2-R_{r2}^2-(R_a+h_2)^2}{-2R_{r2}(R_a+h_2)} \tag{44}$$

$$\sin\theta_{2r} = \frac{R_a}{R_a+h_2}\cos\theta_g \tag{45}$$

Defining the transmitting antenna gains for the direct and reflected paths respectively as G_{td}, G_{tr} and the receiving antenna gains for the direct and reflected paths respectively as G_{rd}, G_{rr} the expressions for these gains can be written directly.

$$G_{td} = \frac{\cos\left[\frac{\Pi}{2}\cos\left(\theta_{1d}\right)\right]}{\sin\theta_{1d}} \tag{46}$$

$$G_{tr} = \frac{\cos\left[\frac{\Pi}{2}\cos\left(\theta_{1r}\right)\right]}{\sin\theta_{1r}} \tag{47}$$

$$G_{rd} = \frac{\cos\left[\frac{\Pi}{2}\cos\left(\theta_{2d}\right)\right]}{\sin\theta_{2d}} \tag{48}$$

$$G_{rr} = \frac{\cos\left[\frac{\Pi}{2}\cos\left(\theta_{2r}\right)\right]}{\sin\theta_{2r}} \tag{49}$$

j. Path Loss for the Specular Reflection or Interference Region

It has been stated earlier that in the interference region the direct signal is in part cancelled or reinforced by the reflected signal. The controlling factors in this phenomenon

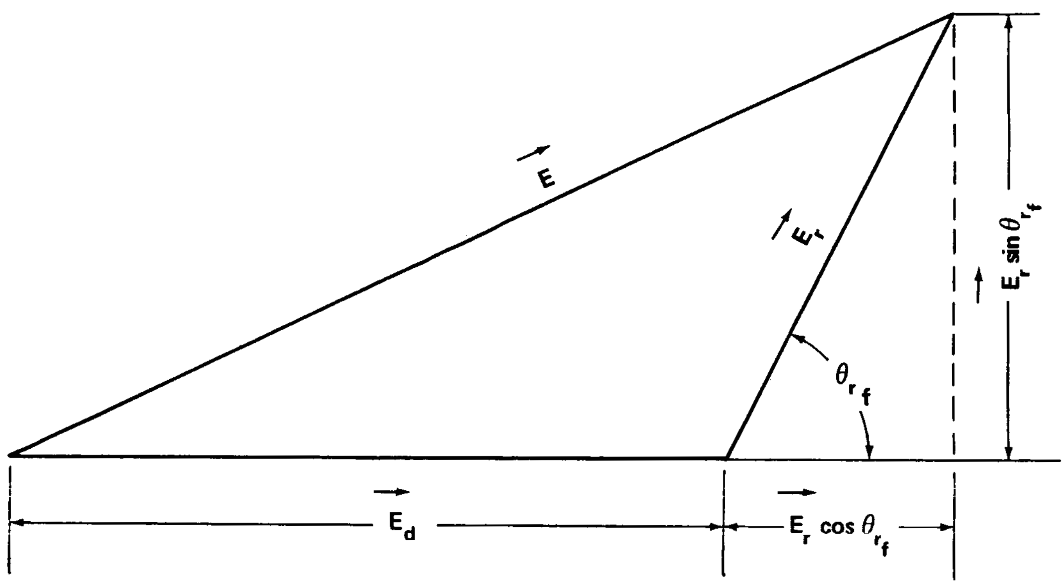


FIGURE 9 - VECTOR DIAGRAM OF RECEIVED SIGNAL (E) IN SPECULAR REFLECTION REGION

are the magnitudes of the two signals and the relative phase between them. This means that the detected signal at the receiver "E" is the vector sum of the direct and the reflected signal present there. Then (see figure 9) also see ref. [6] page 136.

$$E^2 = (E_r \sin \Theta_{rf})^2 + (E_d + E_r \cos \Theta_{rf})^2 \tag{50}$$

$$= E_r^2 + E_d^2 + 2E_d E_r \cos \Theta_{rf} \tag{51}$$

E_d = The direct signal received

$$= \frac{E_o}{R_{fs}} G_{td} G_{rd}$$

E_r = The reflected signal received

$$= \frac{|\Gamma| E_o G_{tr} G_{rr}}{R_{r1}+R_{r2}}$$ with a phase delay of Θ_{rf}

$\frac{E_o}{R_{fs}}$ = The field strength at a receiver, a distance R_{fs} from the transmitter assuming that the transmission medium is free space.

Then for a radio link operating near a rough spherical surface the received signal power in the specular reflection region is

$$E^2 = \left[\frac{|\Gamma_r| E_o G_{tr} G_{rr}}{R_{r1}+R_{r2}}\right]^2 + \left[\frac{E_o}{R_{fs}} G_{td} G_{rd}\right]^2 + \frac{2|\Gamma_r| E_o^2 G_{td} G_{rd} G_{tr} G_{rr} \cos \Theta_{rf}}{R_{fs}(R_{r1}+R_{r2})} \tag{52}$$

The path loss, in excess of that resulting from a free space path R_{fs} meters in length then is given by

$$\left(\frac{\frac{E^2}{E_o}}{R_{fs}}\right)^2 = \left[\frac{|\Gamma_r| G_{tr}\ G_{rr}}{1 + \frac{\delta_{path}}{R_{fs}}}\right]^2 + \frac{2|\Gamma_r| G_{td} G_{rd} G_{tr} G_{rr} \cos \Theta_{rf}}{1 + \frac{\delta_{path}}{R_{fs}}} + G_{td} G_{rd}{}^2 \qquad (53)$$

The total path loss for the specular reflection region then is given by the sum of equation (17) and (53)

$$\text{Path loss (II)} = 20 \log_{10} R_{fskm} + 20 \log_{10} F + 32.45 \qquad (54)$$

$$- 10 \log_{10} \left[\left[\frac{|\Gamma_r| G_{tr} G_{rr}}{1 + \frac{\delta_{path}}{R_{fs}}}\right]^2 + \frac{2|\Gamma_r| G_{td} G_{rd} G_{tr} G_{rr} \cos\Theta_{rf}}{1 + \frac{\delta_{path}}{R_{fs}}} + (G_{td} G_{rd})^2\right]$$

R_{fskm}, R_{fs} = Direct path distance between antennas in kilometers and meters respectively.

k. Bounds on Specular Reflection Region

In section (a), equation (16) above, a constraint was placed on the maximum magnitude of the grazing angle that permits the reflection process to be considered specular. This constraint is repeated here.

$$\Theta_g \leq \sin^{-1} \left[\frac{\lambda}{8\Delta h}\right] \text{ radians} \qquad (16)$$

Let

$$\Theta_{g\ max} = \sin^{-1} \left[\frac{\lambda}{8\Delta h}\right] \qquad (55)$$

There is also a constraint, equation (28), on the minimum magnitude of the grazing angle resulting from the use of the divergence relation. This constraint is repeated here also.

$$\Theta_g \geq \tan^{-1} \left[\frac{\lambda}{2\pi R_a}\right]^{1/3} \text{ radians} \qquad (28)$$

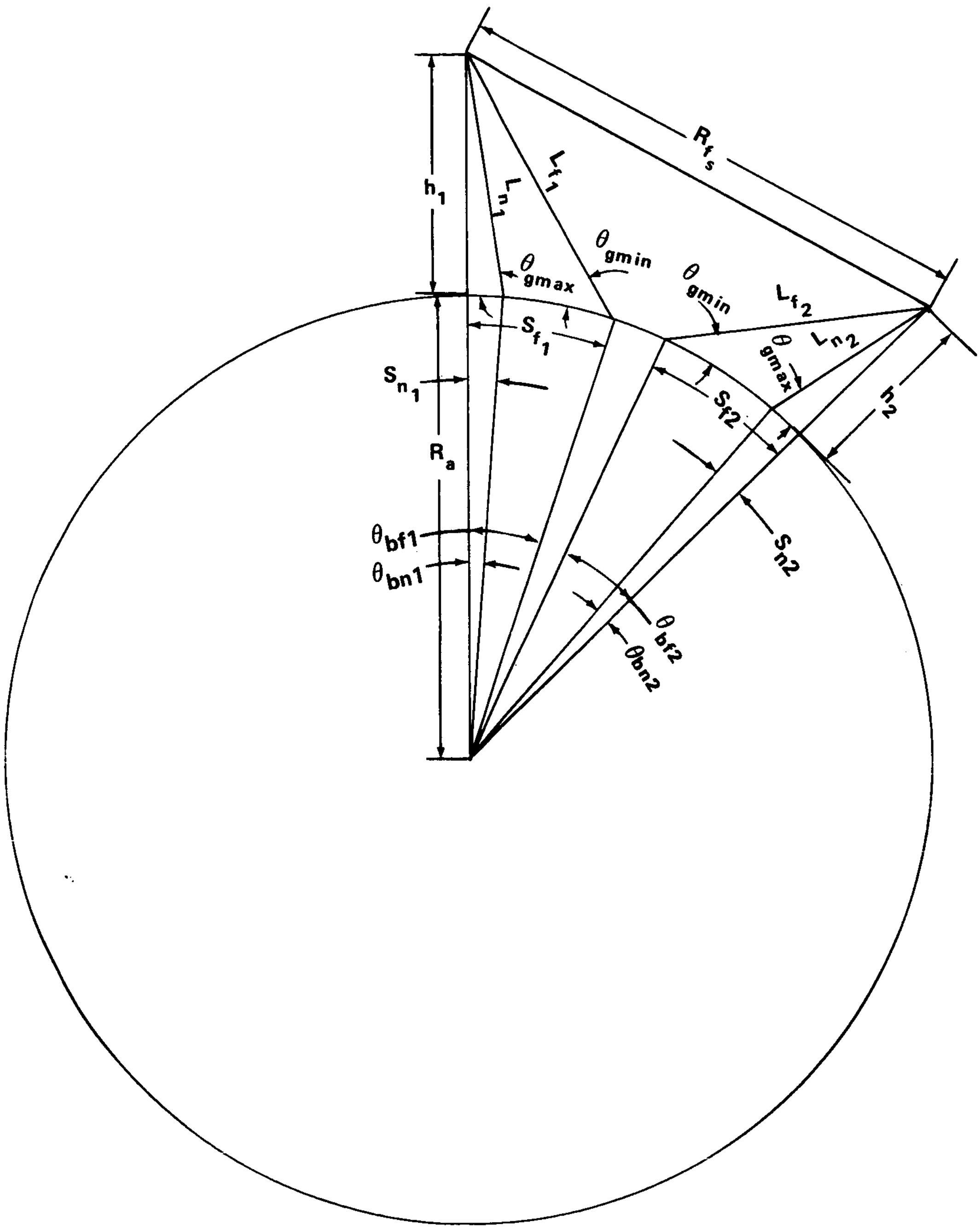


FIGURE 10 – BOUNDS ON SPECULAR REFLECTION REGION

Let

$$\Theta_{g\,min} = \tan^{-1} \left[\frac{\lambda}{2\pi R_a}\right]^{1/3} \tag{56}$$

To put this constraint in a more useable form we convert these bounds to surface distances between the antennas. (See figure 10).

Using the law of cosines

$$\left(R_a+h_1\right)^2 = \left(R_a^2+L_1^2-2R_aL_1 \cos\left(\frac{\pi}{2}+\Theta_{gm}\right)\right) \tag{57}$$

where $L_1 = L_{f1}$, or L_{n1}

$$= R_a^2+L_1^2+2R_aL_1 \sin \Theta_{gm}$$

Using the quadratic formula:

$$L_1^2 + L_1 \left[2R_a \sin \Theta_{gm}\right] + R_a^2 - \left(R_a+h_1\right)^2 = o$$

$$L_1 = \frac{-2R_a\sin\Theta_{gm} \pm \left\{4R_a^2\sin^2\Theta_{gm} - 4\left[R_a^2 - (R_a+h_1)^2\right]\right\}^{1/2}}{2}$$

$$= - R_a\sin\Theta_{gm} \pm \left\{R_a^2\sin^2\Theta_{gm} + 2R_ah_1+h_1^2\right\}^{1/2}$$

$$= R_a \left\{- \sin\Theta_{gm} \pm \left[\sin^2\Theta_{gm} + \frac{2h_1}{R_a} + \frac{h_1^2}{R_a^2}\right]^{1/2}\right\} \tag{58}$$

Since L_1 is a distance as shown in figure 10 and can never be negative, the positive sign in front of the radical must be used.

$$L_1 = R_a \left\{ - \sin\Theta_{gm} + \left[\sin^2\Theta_{gm} + \frac{2h_1}{R_a} + \frac{h_1^2}{R_a^2} \right]^{1/2} \right\} \tag{59}$$

Now knowing L_1 the central angle is calculated.

$$L_1^2 = (R_a+h_1)^2 + R_a^2 - 2(R_a+h_1)R_a\cos\Theta_B$$

$$\cos\Theta_B = \frac{L_1^2-(R_a+h_1)^2-R_a^2}{-2(R_a+h_1)R_a} \tag{60}$$

In general the bounds in surface distance are given by

$$\text{Bound} = R_a\ \Theta_B \text{ in meters} \tag{61}$$

where the nearest bound is

$$\text{Bound}_{(near)} = R_a \cos^{-1}\left[\frac{L_{n1}^2-(R_a+h_1)^2-R_a^2}{-2(R_a+h_1)R_a}\right] + R_a \cos^{-1}\left[\frac{L_{n2}^2-(R_a+h_2)^2-R_a^2}{-2(R_a+h_2)R_a}\right] \tag{62}$$

$$L_{n1} = R_a \left\{ - \sin\Theta_{g\ max} + \left[\sin^2\Theta_{g\ max} + \frac{2h_1}{R_a} + \frac{h_1^2}{R_a^2} \right]^{1/2} \right\}$$

$$L_{n2} = R_a \left\{ - \sin\Theta_{g\ max} + \left[\sin^2\Theta_{g\ max} + \frac{2h_2}{R_a} + \frac{h_2^2}{R_a^2} \right]^{1/2} \right\}$$

and the furthest bound is

$$\text{Bound}_{(\text{far})} = R_a \cos^{-1}\left[\frac{L_{f1}^2 - (R_a+h_1)^2 - R_a^2}{-2(R_a+h_1)R_a}\right] \tag{63}$$

$$+ R_a \cos^{-1}\left[\frac{L_{f2}^2 - (R_a+h_2)^2 - R_a^2}{-2(R_a+h_2)R_a}\right]$$

$$L_{f1} = R_a\left\{- \sin\Theta_{g\ min} + \left[\sin^2\Theta_{g\ min} + \frac{2h_1}{R_a} + \frac{h_1^2}{R_a^2}\right]^{1/2}\right\}$$

$$L_{f2} = R_a\left\{- \sin\Theta_{g\ min} + \left[\sin^2\Theta_{g\ min} + \frac{2h_2}{R_a} + \frac{h_2^2}{R_a^2}\right]^{1/2}\right\}$$

V. PATHLOSS FOR THE REGION NEAR AND BELOW THE HORIZON

a. General Relation for Signal Strength

Near the horizon and below it, the received signal is comprised essentially of the surface wave that can be calculated using the residue series analysis by Bremmer [2, 6]. The following relations are taken from reference [6].

In general the received signal in this region can be expressed, for vertical antennas, as

$$E = \frac{2E_o}{R_{sd}} A_1 F_s f_{(h1)} f_{(h2)} \tag{64}$$

$\frac{E_o}{R_{sd}}$ = the electric field strength at a distance R_{sd} from the transmitting antenna for a free space transmission path

The following factors will be discussed in more detail below

A_1 = the plane surface factor that is a function of the electrical properties of the surface material

F_s = the shadow factor that is a function of the curvature of the surface

$f(h_{1,2})$ = the antenna height gain factors that are a function of the distance which the antennas are located above the surface

b. Plane Surface Factor

If the surface were plane and perfectly conducting the factor A_1 would equal unity; however, for a real plane surface this factor is given by

$$A_1 = \frac{1}{p' R_{sd}} \tag{65}$$

for vertically polarized waves

$$p' = \frac{2\pi}{\lambda} \frac{|\bar{\varepsilon}_c - 1|}{|\bar{\varepsilon}_c|^2}$$

λ = wavelength of the carrier signal in meters

$\bar{\varepsilon}_c = \varepsilon_r - j60\lambda\sigma$

ε_r = relative dielectric constant of the surface in mhos/meter

R_{sd} = surface distance between antennas

There is a constraint placed on the relation for A_1 given above; reference [6] states that the following must be satisfied

$\lambda \lesssim 10$ meters

and

$R_{sd} \gtrsim \frac{50}{p'}$ if $\lambda > 1$ meter and the vertically polarized wave is transmitted over sea water

c. Shadow Factor

The shadow factor represents the effect of the surface curvature in increasing the path loss over that of a plane surface. This factor is defined as follows:

$$F_s = (2\pi)^{1/2} \quad \zeta^{3/2} \quad \left| \sum_n \frac{e^{-j\tau_n \zeta}}{1 + 2\,\frac{\tau_n}{\delta}} \right| \tag{66}$$

for vertically polarized waves

$$\zeta = \left(\frac{2\pi}{\lambda R_a^2}\right)^{1/3} \quad R_{sd}$$

$$\delta = \left[\frac{2\pi R_a}{\lambda}\right]^{2/3} \quad \left(\frac{\varepsilon_c - 1}{\varepsilon_c^2}\right)$$

τ_n = mode numbers

There appears to be some confusion in the definition of the factor "τ_n" in references [2] and [6]. This confusion, I believe, stems in part from the different definitions used for "δ" in the two references. The following definition is believed to be consistent with the analysis by Bremmer in reference [2].

$$\tau_{o,\infty} = 1.856\ e^{-j\pi/3}$$

$$\tau_{1,\infty} = 3.245\ e^{-j\pi/3}$$

$$\tau_{2,\infty} = 4.382\ e^{-j\pi/3}$$

$$\tau_{n,\infty} = \frac{1}{2} \left[3\pi\left(n + \frac{3}{4}\right)\right]^{2/3} e^{-j\pi/3} \quad \text{for } n \geq 3$$

and finally

$$\tau_n = \tau_{n,\infty} - \delta^{-1/2} - \frac{2}{3}\,\tau_{n,\infty}\,\delta^{-3/2} + \frac{1}{2}\,\delta^{-4/2} - \frac{4}{5}\,\tau_{n,\infty}^2\,\delta^{-5/2} + \ldots$$

d. Antenna Height-Gain Factor

Using the shadow factor and the plane surface factor described previously, the received signal can be defined for the case where the antennas are positioned on the spherical surface. A gain, or, an increase in the magnitude of this received signal, is obtained by elevating the antennas to some height (h) above the surface. This gain is defined for vertically polarized waves by

$$f(h) = 1 + j \left[\frac{2\pi h}{\lambda} \left(\frac{\epsilon_c - 1}{\epsilon_c}\right)^{1/2}\right] \tag{67}$$

This expression for height - gain is valid only for

$$h < 30\,\lambda^{2/3} \tag{68}$$

e. Path Loss Expression for the Region Beyond the Specular Reflection Region

Substituting the appropriate expressions into equation 64, gives the following expression for the path loss, in excess of that resulting from a free space path of length R_{sd},

$$\frac{E^2}{\left(\frac{E_o}{R_{sd}}\right)^2} = \left[\frac{2(2\pi)^{1/2}\zeta^{3/2}}{p' R_{sd}} \left|\sum_n \frac{e^{-j\tau_n\zeta}}{1 + \frac{2\tau_n}{\delta}}\right| \left|f(h_1)\ f(h_2)\right|\right]^2 \tag{69}$$

The total path loss then for the region near and below the horizon is given by the sum of equation 17 and 69.

$$\text{Path loss}_{(III)} = 20 \log_{10} R_{sdkm} + 20 \log_{10} F + 32.45 \qquad (70)$$

$$- 10 \log_{10} \left[\frac{8\pi\zeta^3}{(p' R_{sd})^2} \left| \sum_n \frac{e^{-j\tau_n\zeta}}{1 + \frac{2\tau_n}{\delta}} \right|^2 \left| f(h_1)\ f(h_2) \right|^2 \right] \text{decibels}$$

R_{sdkm}, R_{sd} = Surface distance between antennas in kilometers and meters respectively.

VI. SUMMARY

Revised path loss expressions have been presented that, with their implementation in a computer program, permit rapid and it is believed accurate path loss calculations for a radio link operating near a rough spherical surface.

Using the expressions presented here the path losses calculated are equal to those predicated by Schmid [1] for the EVA-EVA case, but differ from Schmid's predictions for the LM-EVA case. The difference (see figures 3) is, however, limited to the specular reflection region calculations and stems from the use by Schmid of two approximations for the phase delay of the reflected signal -

In the EVA-EVA case

$$\phi \simeq \pi + \frac{4\pi h_1 h_2}{\lambda R_{fs}}$$

In the LM-EVA case

$$\phi \simeq \pi + \frac{\pi}{R_{fs}} \quad \frac{(h_1 + h_2)^2}{\lambda}$$

Based on the computer results obtained now it is believed that the first approximation above is more accurate than the second; however, both may be satisfactory for hand calculating path losses.

The use of path loss expressions derived from Bremmer's residue series analysis does permit the rapid calculation of predicted path losses for a wide range of link configurations; however, the validity of these calculations still remains to be tested by lunar surface communications experience.

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Table I

Summary of Path Loss Equations

I. Diffuse Reflection Region (region near transmitting antenna)

$$\text{Path Loss}_{(I)} = 20 \log_{10} R_{fskm} + 20 \log_{10} F + 32.45 \text{ decibels} \quad (17)$$

R_{fskm} = Direct path distance between antennas in kilometers

F = Frequency of Carrier Signal in MHz

II. Specular Reflection Region (region extending beyond diffuse reflection region to near horizon)

$$\text{Path Loss}_{(II)} = 20 \log_{10} R_{fskm} + 20 \log_{10} F + 32.45$$

$$- 10 \log_{10} \left\{ \left[\frac{|\Gamma_r| G_{tr} G_{rr}}{1 + \frac{\delta_{path}}{R_{fs}}} \right]^2 + \frac{2|\Gamma_r| G_{td} G_{rd} G_{tr} G_{rr} \cos\Theta_{rf}}{1 + \frac{\delta_{path}}{R_{fs}}} + \left[G_{td} G_{rd} \right]^2 \right\} \text{ decibels} \quad (54)$$

Γ_r = Reflection coefficient of rough spherical surface

$= \rho_{rms} |\Gamma_p|$ Div

ρ_{rms} = Rough surface factor

$$= \left\{ \exp \left[- \left(\frac{4\pi\Delta h \sin}{\lambda} \Theta_g \right)^2 \right] \right\}^{1/2}$$

Δh = Standard deviation of the heights of the surface irregularities.

R_{fs} = Direct path distance between antennas in meters.

Table I (con'd.)

III. Diffraction Region (region near and below horizon)

$$\text{Path Loss}_{(III)} = 20 \log_{10} R_{sdkm} + 20 \log_{10} F + 32.45$$
$$- 10 \log_{10} \left\{ \frac{8\pi\zeta^3}{\left|p' R_{sd}\right|^2} \left| \sum_n \frac{e^{-j\tau_n \zeta}}{1 + \frac{2\tau_n}{\delta}} \right|^2 \left| f_1(h_1) f(h_2) \right|^2 \right\} \quad (70)$$

R_{sdkm}, R_{sd} = Surface distance between antennas in kilometers and meters respectively.

ζ = Distance factor

$$= \left(\frac{2\pi}{\lambda R_a^2}\right)^{1/3} R_{sd}$$

$$\delta = \left[\frac{2\pi R_a}{\lambda}\right]^{2/3} \left(\frac{\bar{\varepsilon}_c - 1}{\bar{\varepsilon}_c^{\,2}}\right)$$

$$p' = \frac{2\pi}{\lambda} \frac{\left(\bar{\varepsilon}_c - 1\right)}{\left|\bar{\varepsilon}_c\right|^2}$$

$f(h_{1,2})$ = antenna height-gain factors

$$= 1 + j\left[\frac{2\pi h_{1,2}}{\lambda}\right] \frac{\left(\bar{\varepsilon}_c - 1\right)^{1/2}}{\bar{\varepsilon}_c}$$

τ_n = Mode numbers (see equations in text)

Table I (cont'd.)

Θ_g = Grazing angle of the surface reflected signal

λ = Wavelength in meters of the carrier signal

Γ_p = Plane surface reflection coefficient

$$= \frac{\bar{\varepsilon}_c \sin \Theta_g - \left[\bar{\varepsilon}_c - \cos^2 \Theta_g\right]^{1/2}}{\bar{\varepsilon}_c \sin \Theta_g + \left[\bar{\varepsilon}_c - \cos^2 \Theta_g\right]^{1/2}} = |\Gamma_p| \angle \Theta_\Gamma$$

$\bar{\varepsilon}_c = \varepsilon_r - j60\sigma\lambda$

ε_r = Relative dielectric constant of the surface

σ = Conductivity of the surface in mhos/meter

Div = Divergence of the spherical surface

$$= \left\{1 + \frac{2R_{r1}R_{r2}}{R_a\left[R_{r1}+R_{r2}\right] \sin \Theta_g} \left[1+\sin\Theta_g + \frac{2R_{r1}R_{r2}}{R_a\left[R_{r1}+R_{r2}\right]}\right]\right\}^{-1/2}$$

(see figure 8 for definition of terms)

δ_{path} = Difference in path length between the direct and reflected paths in meters

$$\Theta_{rf} = \Theta_\Gamma + \frac{2\pi\delta_{path}}{\lambda}$$

G_{td}, G_{tr} = Gain of transmitting antenna for direct and reflected signals respectively

G_{rd}, G_{rr} = Gain of receiving antenna for direct and reflected signals respectively

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